Average cost and marginal cost

Demonstrate that when the average cost is decreasing, the marginal cost is less than the average cost.

Solution

Recall the definitions of average cost and marginal cost:

$$CM_e = \frac{C(x)}{x}$$
$$CM_g = \frac{dC(x)}{dx}$$

If the average cost is decreasing, it means

$$\frac{dCM_e}{dx} < 0.$$

Therefore,

$$\frac{dCM_e}{dx} = \frac{\frac{dC(x)}{dx} \cdot x - C(x)}{x^2} = \frac{1}{x}\left(\frac{dC(x)}{dx} - \frac{C(x)}{x}\right).$$

Then,

$$\frac{dCM_e}{dx} < 0 \iff \frac{dC(x)}{dx} - \frac{C(x)}{x} < 0$$
$$\frac{dC(x)}{dx} < \frac{C(x)}{x}$$

which means the marginal cost is less than the average cost.